Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 31 (2012) 922 – 927

Procedia Engineering

www.elsevier.com/locate/procedia

International Conference on Advances in Computational Modeling and Simulation

Exterior Trajectory Computation and Modeling of Kinetic Energy Ammunition Based on Constant Terminal Effect

Xiao-jun Zhai, Xu-dan Dong*, Wei Luo

Department of Equipment and Transportation Engineering College of CAPF, Xi'an 710086,China

Abstract

In order to get the corresponding relationship of target distance and the initial speed of muzzle at the moment of non-lethal blow of constant terminal effect, a certain type of 9mm pillar rubber kinetic energy ammunition is taken as an example. The Mutilating physiological effect experiment and numerical fitting are conducted to comprehensively analyze the Efficiency value when being struck. With exterior trajectory equation, compute the exterior trajectory according to Siacci Method and validate with simulation. Then polynomial fit modeling is applied to the calculated statistics to get the simple empirical equation corresponding to the target distance and the initial speed of the muzzle. Based on this, the father interior ballistics design of non-lethal blow of constant terminal effect and energy control experiment of kinetic energy ammunition could be well done, and the exterior trajectory models and empirical equations of other types of kinetic energy ammunitions could be calculated.

*keywords:*constant terminal effect; exterior trajectory; computation; modeling

1. Introduction

When police executes anti-terrorism task, main rioters appear inconstantly and the location is changing continuously. In order to effectively combat with rioters, control development of the situation, special requirements on weapon equipment are proposed. Killing power is adjustable, and terminal performance is

* Corresponding author.Tel:+86-15991894731;
E-mail address:dongxudan@163.com.

1877-7058 © 2011 Published by Elsevier Ltd.
doi:10.1016/j.proeng.2012.01.1122

controllable. The current improved design is on basis of existing domestic riot gun. According to the distance of target, opening in the appropriate parts of barrel is to vent out right amount of gunpowder gas, adjusting kinetic energy projectile muzzle velocity is to achieve constant terminal efficiency non fatal blow of different distant target.

The basis of adjusting kinetic energy projectile muzzle velocity is accurately obtained the relationship between target distance and muzzle velocity. The paper takes a 9mm cylindrical rubber kinetic energy projectile for example, and establishes outside trajectory calculation mathematical model to deduce empirical formula about the relationship between target distance and muzzle velocity, which can lay the foundation for internal ballistic analysis.

1. Determine the constant terminal performance value

Riot kinetic energy weapon combat against staff who expose skin and wear a small amount of clothes, projectile directly effect on target skin, universal threshold can be extracted with kinetic energy for the standard. It should adopt the specific kinetic energy injury criterion [1]. While striking target, the ratio between kinetic energy of the projectile on the contact with the maximum area of the target sectional surface parallel to the projectile is specific kinetic energy (E_d).

Firstly, the damage physiological effect experiment carries out. The test uses biological targets, including pigs, leg of pork, pork chops, etc. The experimental device is the United States miro3 type high speed camera and XGK-2002 small target speed measuring laser screen target. The ballistic gun is fixed, and kinetic energy projectile launch platform is established. By adjusting loading dose and different target distance, the different target speeds are received to achieve combating against the layer skin of biological targets in different specific kinetic energy [2]. The analysis results are as follow Table 1.

Table 1. penetrate depth of the projectile of different specific kinetic energy

specific kinetic energy E_d (J/cm^2)	≤7	8	9	10	11	12	13
penetrate depth H (mm)	0	0	0.2	0.6	1.2	1.9	2.9

Combined damage physiological effect experiment with foreign relevant experimental results [3-4], the analysis is that when the specific kinetic energy reached to 10J/cm^2 left or right, the projectile was bounced, but began to invade the skin of biological targets and just broke it. The hit parts are tumid and slightly damaged, which is consistent with mild damage standard. Considering nonlethal effects and lethality of the kinetic energy projectile, when blow constant terminal kinetic energy is 10J/cm^2, the projectile can make the target loss resistance instantly without causing greater harm. Therefore, when the constant terminal performance is 10J/cm^2, the projectile velocity is calculated, v =79.740m/s.

2. Computation of Exterior Trajectory

Given the range and kinetic energy of projectile are not too high, the whole movement is the movement near the surface, do assumptions: the projectile is axisymmetric, the nutation angle $\delta = 0$; as the small range, do not take the curvature of the earth and changes of the acceleration due to gravity with height (g=9.8m/s^2, and the direction is always vertical down);Coriolis acceleration $a_c = 0$;standard meteorological conditions: no wind, rain and snow. Under these assumptions, the forces acting on the projectile only are gravity and

air resistance, regard the projectile as a particle whose mass being concentrated in the center. Get the vector equation of the motion:

$$\frac{dv}{dt} = a_x + g \tag{1}$$

Project the vector equation (1) on the horizontal direction, then get the exterior trajectory equation:

$$\begin{cases} \dfrac{du}{dx} = -CH(y)G(v) & (2) \\ \dfrac{d\theta}{dx} = -\dfrac{g}{v^2} & (3) \\ \dfrac{dy}{dx} = \tan\theta & (4) \\ \dfrac{dt}{dx} = \dfrac{1}{u} & (5) \end{cases}$$

Thereinto,

(x,y)——coordinate of the projectile at any trajectory point; v——the speed of the projectile; C——Ballistic coefficient; $H(y)$——a function of air density, low-stretch ballistic standard conditions: $H(y)=1$; $G(v)$——air resistance function; $G(v)=4.737\times10^{-4}vC_{x0N}(M)$, $C_{x0N}(M)$ as the standard missile drag coefficient; g——acceleration due to gravity, $g = 9.8\text{m/s}^2$; θ——angle between projectile axis with the horizontal at any trajectory point.

By Siacci Method, any point on the trajectory there is $\theta_0 > \theta$, the introduction of $U(U = \dfrac{u}{\cos\theta_0})$ instead of $v(v = \dfrac{u}{\cos\theta})$,separate variables u with θ in $G(\dfrac{u}{\cos\theta})$, then $U > v$. For the trajectory with small angle ($\theta_0 < 5°$), regard as $\cos\theta_0 \approx \cos\theta$, then error between $G(U)$ and $G(v)$ is smaller, $G(U)$ is an increasing function of U, $G(v)<G(U)$, for the introduction of close substitutes, with $\cos\theta_0$ to further correct the error. That $G(v) = \cos\theta_0 \cdot G(U)$, by the equation (2), $\dfrac{dU}{U} = -4.737\times10^{-4}\, CC_{x0N}(M)dx$, That $k = 4.737\times10^{-4}\, CC_{x0N}(M)$, then $\dfrac{dU}{U} = -kdx$, Which, $U_0 = v_0$, $U = \dfrac{\cos\theta}{\cos\theta_0} v$, We get $v = \dfrac{\cos\theta_0}{\cos\theta} v_0 e^{-kx}$ (6), take (6) into (3), $\dfrac{d\theta}{\cos^2\theta} = -\dfrac{ge^{2kx}}{\cos^2\theta_0 v_0^2}dx$,We get $\tan\theta = \dfrac{-g(e^{2kx}-1)}{2kv_0^2\cos^2\theta_0} + \tan\theta_0$ (7), compare (6) and (7), then

$$\begin{cases} v = \dfrac{\cos\theta_0}{\cos\theta} v_0 e^{-kx} \\ \tan\theta = \dfrac{-g(e^{2kx}-1)}{2kv_0^2\cos^2\theta_0} + \tan\theta_0 \end{cases}$$

Table 2. The desired muzzle velocity corresponding to target distance

i	target distance x_i (m)	muzzle velocity v_{0i} (m/s)
1	10	88
2	20	97
3	30	107

4	40	118
5	50	130
6	60	144
7	70	160
8	80	177
9	90	195

For constant kinetic energy weapons, when firing with anti-riot nonfatal blow, effective range is no more than 100m, for the 9mm rubber cylindrical projectiles, m=2g, v_0=230m/s, d=9mm, i=2, θ_0=π/170. v, θ_0, x, C is constant, we can find v_0. Calculated data are shown in Table 2.

3. Trajectory Simulation

With the exterior trajectory equation, use integration modules when establishing differential equations, determine the initial value according to the initial conditions. Establish simulation model with Math Operation Module group and Integrator module of Continuous Module group.Display the simulated ballistic curve and speed curve with the distance with the X-Y Graph block. Simulation model is diagrammed in Fig. 1. (a). Write M function file, set the simulation module parameter values with the function set_param(); execute Simulink model by sim command. Set the initial velocity from velocity v0i in Table 2; use adaptive variable step integration method of ode15s for simulation; with allowed relative error range of 1e-10; start from time 0; end at the corresponding target distance. When velocity v_{0i} less than 150m/s, $C_{x0N}(M)$=0.255; velocity v_{0i} higher than 150m/s, $C_{x0N}(M)$=0.259. Respective Simulation curves of speed and the corresponding range are shown in Fig. 1. (b), Fig. 2. (a).

The projectile velocity as in Table 2 obtained $v_{0i,}$ in the corresponding target distance, the target speed can achieve 80m/s with a little smaller error.The simulation results can reach the desired requirements, and comparing with theoretical computation values, it is similar to a high degree of numerical results. Therefore, this external ballistics solution method and computational results are correct.

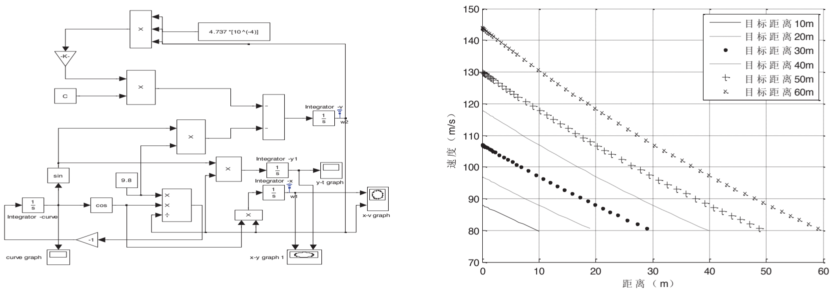

Fig. 1. (a) Simulation model;(b) simulation curves (v_{0i}<150m/s）

4. Polynomial Fitting Model

For the data from computational results (x_i, v_{0i}) (i=1,2,...,9), find a Polynomial m (m <9), p_m(x)= a_0 + a_1x + ... $a_m x^m$, so $\sum_{i=1}^{9}\delta_i^2 = \sum_{i=1}^{9}[v_i - p_m(x_i)]^2 = f(a_0,\cdots a_m)$ (8) minimum. that is, select a_i (i = 0,1,...m),to make $f(a_0,a_1,\cdots a_m) = \min_{\psi\in H}\sum_{i=1}^{9}[v_i - \psi(x_i)]^2$ (9), where H is the group of polynomials up to m times. Satisfy

$\frac{\partial f}{\partial a_j} = -2\sum_{i=1}^{9}\left[v_i - \sum_{k=0}^{m} a_k x_i^k\right]x_i^j = 0$ (j=0,1,⋯m) ,transpose, $\sum_{k=0}^{m} a_k (\sum_{i=1}^{9} x_i^{k+j}) = \sum_{i=1}^{9} v_i x_i^j$ (j=0,1,⋯m) (10). As a result, data processing, model fitting, polynomial curve fitting obtained as Fig. 2.(b),

$p_1(x)$=1.333x+68.444 (11)

$p_2(x)$=0.007x^2+0.628x+81.381 (12)

It can be seen from curve of fitting, the result of polynomial fitting 3 or 4 times is coincident with fitting 2 times. Considering the fitting accuracy and computing complexity, the kinetic energy for the 9mm projectiles, external ballistics modeling take m=2 for the best. For $p_2(x)$,verify the calculated: $\varepsilon_r{}^*{}_i = \left|\frac{p_2(x_i) - v_{0i}}{v_{0i}}\right| < 0.5\%$, ($i$=1,2,…9)

Accuracy of data fitting model meet the actual demand, so the empirical formula about distance and muzzle velocity: $p_2(x)$=0.007x^2+0.628x+81.381

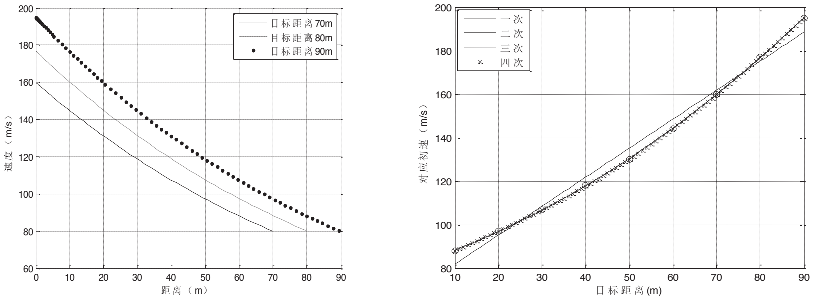

Fig. 2. (a) simulation curves (v_{0i}>150m/s);(b) curve of polynomial fitting

5. Conclusions

(1) Get the corresponding relationship of target distance and the initial speed of muzzle at the moment of non-lethal blow of constant terminal effect. Based on this, the father interior ballistics design of non-lethal blow of constant terminal effect and energy control experiment of kinetic energy ammunition could be well done.

(2) This method for modeling on other non-lethal anti-riot low kinetic energy projectiles is generally applicable. There is a larger value for the computation of exterior trajectory models of other types of kinetic energy ammunitions.

References

[1] RAN Jing-lu, XU Cheng, NIU Fu-qiang. Principle of Variable Muzzle Velocity Shoot for Steady Effectiveness Nonlethal Killing[J].*Journal of Nan jing University of Science and Technology(Natural Science)*, 2010,34(6):765-769.

[2] AN Bao-lin, HE Lei, ZHOU Ke-dong, XU Cheng. Experimental Research of Terminal Effect of Low Velocity Bullets on Live Target[J].*Journal of Ballistics*, 2010,22(3):78-80

[3] ALI M M. Rupture of human skin membrane under impact of paraboloidal projectile: bullet wound ballistics[J]. *Defence Science Journal*, 1996, 46(4):233-246

[4] WATKINS F P, PEARCE B P, STAINER M C. Assessment of terminal effects of high velocity projectiles using tissue stimulants [J]. *Acta Chir Scand*, 1982, 508:39-47

[5] ZHAO Fa-dong, ZHUANG Hong-wei. Matlab/Simulink Based Trajectory Simula tion of A anti-riots Kinetic Energy Munition[J]. *Equipment Manufactring Technology*, 2009,10:51-53